

APEEJAY SCHOOL, SHEIKH SARAI  
FIRST TERM EXAMINATION, 2019–20

SS-45

CLASS-XII  
MATHEMATICS

Time allowed : 3 Hrs.

M.M. : 80

**General Instructions :**

- (i) Questions no. 1 to 20 carry one mark each.
- (ii) Question no. 21 to 26 carry 2 marks each.
- (iii) Question no. 27 to 32 carry 4 marks each.
- (iv) Questions no. 33 to 36 carry 6 marks each.

1. If for a square matrix of order 3,  $|A \text{ adj. } A| = 8$ , Find  $|A|$ .
2. For what value of  $x$ , the matrix  $\begin{bmatrix} 8-x & x+4 \\ 1 & 5 \end{bmatrix}$  is singular ?
3. Evaluate  $\int e^{3 \log x} (x^4) dx$ .
4. Give an example to show that the relation  $R$  in the set of natural numbers defined by  $R = \{(x, y), x, y \in N, x \leq y^2\}$  is not symmetric.
5. Write the principal value branch of  $\operatorname{cosec}^{-1} x$  defined on the domain  $R - \{-1, 1\}$ .
6. If  $A$  is a matrix of order  $2 \times 3$  and  $B$  is a matrix of order  $3 \times 5$ , which is order  $(AB)^t$  ?
7. Evaluate  $\cos^{-1} \left( \cos \frac{13\pi}{6} \right)$ .
8. Find  $(AB)^t$  if  $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ ,  $B^t = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$ .
9. Evaluate  $\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$
10. Differentiate  $e^{\sqrt{\alpha \cos x + 3}}$ .
11.  $\tan^{-1} 2 + \tan^{-1} 3$  is equal to
  - (a)  $\frac{-\pi}{4}$
  - (b)  $\frac{3\pi}{4}$
  - (c)  $\frac{5\pi}{4}$
  - (d)  $\frac{7\pi}{4}$

P.T.O.

12. A singular matrix is invertible, state true or false.

13. Given a  $2 \times 2$  matrix  $A = [a_{ij}]$  where  $a_{ij} = \frac{(i-2j)^2}{3}$  then  $a_{21}$  is

(a) 0 (b)  $\frac{1}{3}$

(c) 3 (d)  $\frac{2}{3}$

14. For what value of  $k$ , the functions :

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & \text{if } x \neq 1 \\ 4k, & \text{if } x = 1 \end{cases} \text{ is continuous at } x = 1 \}$$

(a) 2 (b) 1

(c) 4 (d)  $\frac{1}{2}$

15. Derivative of  $x^x$  with respect to  $x$  is :

(a)  $(1 + \log x)$  (b)  $x \cdot x^{x-1}$

(c)  $x \log x$  (d)  $x^x (1 + \log x)$

16. If  $e^x \cdot y = 1$ , then  $e^x y'' = 1$ , state true or false.

17. Rate of change of values of a sphere of diameter  $r$ , with respect to  $r$  is

(a)  $4\pi r^2$  (b)  $\frac{4}{3}\pi r^2$

(c)  $\frac{1}{2}\pi r^2$  (d)  $\pi r^2$

18. If  $I$  is an Identity matrix  $A$  is a square matrix of the same order such that  $A^2 = A$ , then the value of  $(I + A)^2 - 3A$  is.

(a)  $3I$  (b)  $2I$  (c)  $I$  (d)  $A$

19. The value of determinant  $\begin{vmatrix} \frac{1}{a} & 1 & bc \\ \frac{1}{b} & 1 & ca \\ \frac{1}{c} & 1 & ab \end{vmatrix}$  is

(a)  $a^{-1} b^{-1} c^{-1}$  (b)  $abc$

(c) 0 (d)  $a + b + c$

20. If  $y = \tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}$ , then  $\frac{dy}{dx}$  is equal to

(a)  $\frac{1}{4}$

(b)  $\frac{1}{2}$

(c) 1

(d) 2

21. Prove that  $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 5) = 31$ .

OR

Evaluate  $\cos^{-1}\left(\frac{\cos 11\pi}{6}\right)$

22. If  $y = \left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2$  find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{6}$ .

23. Find the point on the curve  $y = \frac{1}{x-1}$  where tangent has slope equal to 2.

OR

The side of a square is increasing at the rate of 4 cm/min. at what rate is the perimeter increasing when side is 4 cm long.

24. Evaluate :  $\int e^{\sqrt{x}} dx$

OR

Evaluate  $\int \frac{dx}{\sin^2 x \cos^2 x}$

25. Evaluate :  $\int \frac{\sqrt{x} dx}{x+1}$

OR

Evaluate :  $\int \frac{xe^x + e^x}{\sin^2(xe^x)} dx$

26. A cube is expanding such that its edge is changing at the rate of 5 cm/sec. if the edge is 4 cm long, then find the rate of change of its volume.

27. Let  $f : N \rightarrow R$  be a function, defined as  $f(x) = 4x^2 + 12x + 15$ . Show that  $f : N \rightarrow \text{Range of } f$  is invertible also find  $f^{-1}$ .

28. Using properties of determinants, solve for  $x$  :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

OR

Show that  $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

29. A square piece of Tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. Find the maximum volume of the box.

30. Evaluate :  $\int \frac{\log x}{(x+1)^2} dx$

OR

Evaluate :  $\int \frac{dx}{1+x^4}$

31. Verify Rolle's theorem for

$$f(x) = x(x+3)e^{-x/2} \text{ in } [-3,0]$$

32. Water is dripping out from a conical funnel of semi vertical angle  $\pi/6$  at the uniform rate of  $6 \text{ cm}^2/\text{sec}$ , in its curved surface area through a tiny hole at vertex in the bottom when slant height of water is 4 cm, find the rate of decrease of start height of water.

OR

The sum of the length of the hypotenuse and a side of a right triangle is given show that the area of triangle is maximum when the angle between them is  $\pi/3$ .

33. If  $x^m y^n = (x+y)^{m+n}$ , show that  $\frac{dy}{dx} = \frac{y}{x}$ .

OR

If  $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$  find  $\frac{dy}{dx}$ .

34. (i) Check the continuity of  $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  at  $x = 0$

(ii) Differentiate  $\tan^{-1} \left\{ \frac{\sqrt{1+x^2}-1}{x} \right\}$  w.r. to  $x$ .

35. Use product  $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$  to solve the system of equations

$$x - y + 2z = 1$$

$$2y - 3z = 1$$

$$3x - 2y + 4z = 2$$

OR

Solve by matrix method :

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

36. Find  $\frac{dy}{dx}$  if  $y^x + x^y + x^x = a^b$

OR

Find  $\frac{dy}{dx}$  if  $X = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

**APEEJAY SCHOOL,PANCHSHEEL PARK**

**PREBOARD EXAMINATION (2019 – 20)**

**CLASS 12 (MATHS)**

TIME : 3HR

M.M:80

**GENERAL INSTRUCTIONS:**

1. All questions are compulsory.
2. Please check that question paper contains 36 questions.
3. Questions 1 – 20 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 21 – 26 in Section B are short answer type questions carrying 2 marks each.
5. Questions 27 – 32 in Section C are long-answer-I type questions carrying 4 marks each.
6. Questions 33 – 36 in Section D are long-answer-II type questions carrying 6 marks each.
6. Please write down the serial number of the question before attempting it.

**SECTION A**

1. Give an example of singular matrix.
2. If for matrix A,  $|A| = 5$ , find  $|4A|$ , where A is of order  $2 \times 2$ .
3. If the lines  $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{3}$  and  $\frac{x-2}{4p} = \frac{y-5}{4} = \frac{z-1}{-7}$  are perpendicular to each other, then find the value of p.
4. If A and B are two events such that  $P(A)=0.2$ ,  $P(B)=0.4$  and  $P(A \cap B)=0.8$  then
  - a) A, B are mutually exclusive.
  - b) A, B are exhaustive.
  - c) A, B are independent.
  - d) none of these.
5. The point (2,1) lies in the half plane  $2x+3y-12 \leq 0$ 
  - a) True
  - b) False
6. Evaluate:  $\cos(2\cos^{-1}(-\frac{3}{5}))$
7. Two persons A,B fire a target in turn there probabilities of hitting the target are 0.4 and 0.6 respectively, the probability that target hit is \_\_\_\_.
8. Evaluate  $\int \frac{dx}{\sqrt{x^2+4}}$
9. For non zero vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  if  $\vec{a} \times \vec{b} = \vec{c} \times \vec{a}$ , then  $\vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$ .
  - a) True
  - b) False

10. y intercept made by the plane  $3x - y + 4z = 9$  is \_\_\_\_\_.

11. The relation R in the set A of all the lines in a plane defined as  $R = \{l_1, l_2 \in A \times A; \text{ is parallel to } l_2\}$  is an equivalence relation. Write the equivalence class related to the line  $2y = 5x + 7$ .

12. For what value of k, the function  $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ 4k, & \text{if } x = 1 \end{cases}$  is continuous at  $x = 1$ .

13. Given a matrix  $A = \begin{bmatrix} 3 & 2 \\ 4 & 4 \end{bmatrix}$ . Then  $|A^{-1}|$  is :

a)  $\begin{bmatrix} 4 & -2 \\ -4 & 3 \end{bmatrix}$

b)  $\frac{1}{4} \begin{bmatrix} 4 & -2 \\ -4 & 3 \end{bmatrix}$

c)  $\frac{1}{4}$

d) 4

14. If tangent to the curve  $y^2 + 3x - 7 = 0$  at the point (h, k) is parallel to line  $x - y = 4$ , then the value of k is \_\_\_\_\_?

15. Find the angle between vectors  $\vec{a}$  and  $\vec{b}$  if  $|\vec{a}| = 1$ ,  $|\vec{b}| = 2$  and  $\vec{a} \times \vec{b} = \hat{i} + \hat{j} + \hat{k}$ .

16. Given a  $2 \times 2$  matrix  $A = [a_{ij}]$ , where  $a_{ij} = \frac{(i-2j)^2}{3}$ . Then  $a_{21}$  is :

a) 0

b)  $\frac{1}{3}$

c) 3

d)  $\frac{2}{3}$

17. Evaluate  $\int_{-2}^2 (x^3 + 1) dx$

18.  $\int_{-1}^1 \log \left| \frac{2-x}{2+x} \right| dx$  is equal to \_\_\_\_\_.

OR

Find  $\int \frac{dx}{1+\sqrt{x}}$

19. Find  $\int x e^{(x+1)} dx$ .

20. Form the differential equation of the family of the curve  $y = a \cos(b + x)$  where a, b are arbitrary constants

21. Prove that  $2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$

SECTION B

22. Solve the differential equation:  $\frac{dy}{dx} = \frac{\sqrt{x^2+4}}{y}$

23. A particle moves along the curve  $x^2 = 2y$ . At what point, ordinate increases at the same rate as abscissa increases?
24. If  $\vec{a} = \hat{i} - \hat{k}$ ,  $\vec{b} = 3\hat{i} - \hat{k}$ ,  $\vec{c} = 7\hat{i} - \hat{j}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$ .  $|\vec{d}| = 1$ .
- OR
25. Find the acute angle between the lines  $\frac{x-4}{3} = \frac{y+3}{4} = \frac{z+1}{5}$  and  $\frac{x-1}{4} = \frac{y+1}{-3} = \frac{z+10}{5}$
26. A speaks truth in 80% cases and B speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact?

### SECTION C

27. Let  $f: A \rightarrow B$  be a function defined as  $f(x) = \frac{2x+3}{2x-3}$  where  $A = \mathbb{R} - \{3\}$  and  $B = \mathbb{R} - \{2\}$ . Is the function  $f$  one-one and onto? Is  $f$  invertible? If yes, then find its inverse.
28. Find the approximate change in the values of  $\frac{1}{x^2}$ , when  $x$  changes from  $x = 3$  to  $x = 3.002$ .
29. Solve the differential equation:  $x dy - y dx = \sqrt{x^2 + y^2} dx$
30. Evaluate the following  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$
- OR
- Evaluate  $\int_{-2}^2 (3x^2 - 2x + 4) dx$  as the limit of a sum.
31. A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively are  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$  and  $\frac{2}{3}$ . What is the probability that:
- (i) the problem will be solved?
- (ii) at most one of them will solve the problem?

### OR

- A bag I contains 5 red and 4 white balls and a bag II contains 3 red and 3 white balls. Two balls are transferred from Bag I to Bag II and then one ball is drawn from the Bag II. If the ball drawn from the Bag II is red, then find the probability that one red and one white ball are transferred from the Bag I to the Bag II.
32. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 hours of labour for fabricating and one hour for finishing. Each piece of model B requires 12 hours of labour for fabricating and 3 hours for finishing. The maximum number of labour hours, available for fabricating and for finishing are, 180 and 30 respectively. The company makes a profit of Rs. 8000 and Rs. 12000 on each piece of model A and model B respectively. How many pieces of each model should be manufactured to get maximum profit? Also, find the maximum profit.



### SECTION D

33. If  $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ , then find  $A^{-1}$  and hence solve the system of equations :  
 $3x + 4y + 7z = 14$  ;  $2x - y + 3z = 4$  ;  $x + 2y - 3z = 0$ .

**OR**

Prove the following using the properties of determinants:

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(3abc - a^3 - b^3 - c^3)$$

34. Using integration, find the area bounded by the curve  $4y = x^2$  and the line whose equation is  $x = 2y$ .
35. A farmer wants to construct a circular garden and a square garden in his field. He wants to keep the sum of their perimeters 600 m. Prove that the sum of their areas is the least, when the side of the square garden is double the radius of the circular garden.
36. Find the distance of the point  $3\hat{i} - 2\hat{j} + \hat{k}$  and from the plane  $3x + y - z + 2 = 0$  measured parallel to the line  $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$ . Also, find the foot of the perpendicular from the given point upon the given plane.

**OR**

Find the equation of the plane passing through the line of intersection of the planes  $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = -1$  and  $\vec{r} \cdot (\hat{i} + \hat{j} - 2\hat{k}) = 0$  and passing through the points  $(3, -2, -1)$ . Also find the angle between the two given planes.



ROLL NO.	
NAME	
CLASS & SECTION	

APEEJAY COMMON PRE-BOARD EXAMINATION, 2019-20

08

CLASS-XII

MATHEMATICS

Time allowed : 3 hrs.

Maximum Marks : 80

General Instructions :

- (i) All questions are compulsory
- (ii) Question 1 to Question 20 carry 1 mark each.
- (iii) Question 21 to Question 26 carry 2 marks each.
- (iv) Question 27 to Question 32 carry 4 marks each.
- (v) Question 33 to Question 36 carry 6 marks each

(SECTION-A)

(1 Mark each)

1. If  $A$  and  $B$  are square matrices of order 3 such that  $|A| = -1$  and  $|B| = 4$ . Then what is the value of  $|3(AB)|$ ?  
(a) -12  
(b) -36  
(c) -102  
(d) -108
2. If  $A$  and  $B$  are square matrices of the same order, then  $(A+B)(A-B)$  is Equal to :  
(a)  $A^2 - B^2$   
(b)  $A^2 - BA - AB - B^2$   
(c)  $A^2 - B^2 + BA - AB$   
(d)  $A^2 - BA + B^2 + AB$
3. Let  $A$  and  $B$  be two events such that  $P(A) = 0.6$ ,  $P(B) = 0.2$  and  $P(A/B) = 0.5$ . Then  $P(A'/B')$  equals  
(a)  $1/10$   
(b)  $3/10$   
(c)  $3/8$   
(d)  $6/7$
4. The area of the parallelogram whose adjacent sides are  $\hat{i} + \hat{k}$  and  $2\hat{i} + \hat{j} + \hat{k}$  is  
(a)  $\sqrt{2}$  sq. units  
(b)  $\sqrt{3}$  sq. units  
(c) 3 sq. units  
(d) 4 sq. units

P.T.O.

5. The point which does not lie in the half plane  $2x + 3y - 12 \leq 0$  is
- (a) (1,2) (b) (2,1)  
(c) (2,3) (d) (-3, 2)
6. Let  $\theta = \sin^{-1}(\sin(-600^\circ))$ ; then value of  $\theta$  is
- (a)  $\pi/3$  (b)  $\pi/2$   
(c)  $2\pi/3$  (d)  $-2\pi/3$
7. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability of getting exactly one red ball is
- (a)  $45/196$  (b)  $135/392$   
(c)  $15/56$  (d)  $15/29$

8.  $\int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$  is equal to

- (a) 1 (b) 2  
(c) 3 (d) 4

9. A line makes equal angles with co-ordinate axis, Direction cosines of this line are :

- (a)  $\pm(1,1,1)$  (b)  $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$   
(c)  $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$  (d)  $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

10. If  $f(x) = \begin{cases} mx + 1, & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ , is continuous at  $x = \frac{\pi}{2}$ , then

- (a)  $m=1, n=0$  (b)  $m = \frac{n\pi}{2} + 1$   
(c)  $n = \frac{m\pi}{2}$  (d)  $m = n = \frac{\pi}{2}$

11. If  $f$  be the greatest integer function defined as  $f(x) = [x]$  and  $g$  be the modulus function defined as  $g(x) = |x|$ , then the value of  $g \circ f\left(\frac{-5}{4}\right)$  is .....

12. If  $x = t^2, y = t^3$ , then  $\frac{d^2y}{dx^2} = \dots\dots\dots$

13. If  $A$  is a square matrix such that  $A^2 = I$ , then  $(A - I)^3 + (A + I)^3 - 7A = \dots\dots\dots$

14. The two curves  $x^3 - 3xy^2 + 2 = 0$  and  $3x^2y - y^3 = 2$  cut at an angle  $\dots\dots\dots$

OR

The values of  $a$  for which the function  $f(x) = \sin x - ax + b$  increases on  $R$  are  $\dots\dots\dots$

15. The angle between the vectors  $\hat{i} - \hat{j}$  and  $\hat{j} - \hat{k}$  is  $\dots\dots\dots$

OR

The projection of the vector  $\vec{a} = 2\hat{j} - \hat{j} + \hat{k}$  along  $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$  is  $\dots\dots\dots$

16. Without expanding, show that

$$\Delta = \begin{vmatrix} \operatorname{cosec}^2\theta & \cot^2\theta & 1 \\ \cot^2 & \operatorname{cosec}^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$$

Properties used  $\dots\dots\dots$

17. If  $\int_0^1 \frac{e^t}{1+t} dt = a$ , then find the value of  $\int_0^1 \frac{e^t}{(1+t)^2} dt$  Value is  $\dots\dots\dots$

18.  $\int \frac{\sec^2(\log x)}{x} dx$  Value is  $\dots\dots\dots$

OR

$\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx$  Value is  $\dots\dots\dots$

19. Evaluate :  $\int \frac{dx}{5 - 8x - x^2}$  Value is  $\dots\dots\dots$

20. Find the differential equation representing the curve  $y = cx + c^2$  Equation is  $\dots\dots\dots$

(SECTION-B)

(2 Marks Each)

21. If  $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}, xy < 1$ , then find the value of  $x + y + xy$

OR

If  $f$  is an invertible function,  $f: R \rightarrow R$  defined as  $f(x) = \frac{3x - 4}{5}$ , write  $f^{-1}(x)$ .

22. Evaluate :  $\int \frac{dx}{\sqrt{x^2 + 2x + 2}}$

23. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.

24. If  $|\vec{a}| = a$ , then find the value of the following

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$$

OR

For any two vectors  $\vec{a}$  and  $\vec{b}$ , Prove that  $(\vec{a} \times \vec{b})^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$

25. Find the value of  $p$  so that the lines  $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{5z-10}{11}$  and  $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$  are perpendicular to each other.

26. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event 'number obtained is green' and B be the event 'number obtained is red'. Find A and B are independent events or not.

(SECTION-C)

(4 marks each)

27. Let  $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ . Show that  $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$  is an Equivalence relation, Write the equivalence class [2].

28. Find the equation(s) of the tangent(s) to the curve  $y = (x^3 - 1)(x - 2)$  at the points where the curve intersects the  $x$ -axis.

OR

If  $x^m y^n = (x + y)^{m+n}$ , prove that  $\frac{d^2 y}{dx^2} = 0$

29. Prove that  $x^2 - y^2 = c(x^2 + y^2)^2$  is the general solution of the differential equation  $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$ , where  $c$  is a parameter.

30. Evaluate :  $\int_0^{3/2} |x \sin \pi x| dx$ .

31. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let  $X$  denote the sum of the numbers on the two drawn cards. Find the mean and variance of  $X$ .

OR

Of the students in a school, it is known that 30% have 100% attendance and 70% Students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance?

32. Maximize  $z = x + 2y$

Subject to the constraints

$$x + 2y \geq 100$$

$$2x - y \leq 0$$

$$2x + y \leq 200$$

$$x, y \geq 0$$

Solve the above LPP graphically.

(SECTION-D)

(6 marks each)

33. Using properties of determinants, prove that

$$\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(c+a)^2}{b} \end{vmatrix} = 2(a+b+c)^3$$

OR

If  $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ , find  $A^{-1}$  and hence solve the following system of equations

$$3x + 4y + 7z = 14, 2x - y + 3z = 4, x + 2y - 3z = 0$$

34. Using integration find the area of the region in the first quadrant enclosed by the  $x$ -axis, the Line  $y = x$  and the circle  $x^2 + y^2 = 32$ .

OR

Using integration, find the area of the triangle formed by positive  $x$ -axis and tangent and normal to the circle  $x^2 + y^2 = 4$  at  $(1, \sqrt{3})$ .

35. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, Show that the area of the triangle is maximum, when the angle between them is  $\frac{\pi}{3}$ .
36. Find the equation of the plane through the line of intersection of  $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$  and  $\vec{r} \cdot (\hat{i} - \hat{j}) + 4 = 0$  and perpendicular to the plane  $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$ . Hence find whether the plane thus obtained contains the line  $x - 1 = 2y - 4 = 3z - 12$ .

**BEST OF LUCK!**