APEEJAY SCHOOL, SHEIKH SARAI

FIRST TERM EXAMINATION, 2019–20 **SS-44**

CLASS-XI

MATHEMATICS

Time allowed : 3 Hrs. **M.M. : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The question paper consists of 36 questions divided into four sections A, B, C and D.

(iii) Section-A comprises of 20 questions of 1 mark each. Section-B comprises of 6 questions of 2 marks each Section : C comprises of 6 questions of 4 marks each and Section-D comprises of 4 questions of 6 marks each.

(SECTION : A)

1. Two finite sets have m and n elements. The total number of subsets of the first set is 48 more than the total number of subsets of second set. The value of m and n are :

(a) 7, 6 (b) 6, 3

(c) 7, 4 (d) 6, 4

2. If $A=\{1,2,3,4,5\}$, then the number of proper subsets of A is :

(a) 120 (b) 30

(c) 31 (d) 32

3. In a college of 300 students, every student reads 5 newspapers and every news paper is read by 60 students. The number of newspapers are :

(a) atleast 30 (b) at most 20

(c) Exactly 25 (d) None

4. If two sets A and B are having 99 elements in common, then the number of elements common to each of the sets $A\times B$ and $B\times A$ are

(a) 2^{99} (b) 99^2

(c) 100 (d) 18

5. Let $A=\{1,2,3,4\}$, $B=\{1,2\}$, then total number of function from A to B are :

(a) 16 (b) 15

(b) 2^8 (d) 2^8-1

P.T.O.

6. Let $f : X \rightarrow Y$ be a given function, then f^{-1} exists if

(a) f is one-one
(b) f is onto
(c) f is one-one but not onto
(d) f is one-one and onto

7. If the length of a chord of a circle is equal to that of the radius of a circle then the angle subtended, in radians at the centre of the circle by chord is :

(a) 1
(b) $\frac{\pi}{2}$
(c) π
(d) 2π

8. The maximum value of $\sin\theta\cos\theta$ is :

(a) 1
(b) 0
(c) $-\frac{1}{2}$
(d) $\frac{1}{2}$

9. The value of $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right)$ is

(a) -2
(b) 2
(c) 1
(d) -1

10. The value of $\sin^2 5^\circ+\sin^2 10^\circ+\sin^2 15^\circ+\ldots.. \sin^2 90^\circ$ is

(a) $7\frac{1}{2}$
(b) $8\frac{1}{2}$
(c) $1\frac{1}{2}$
(d) $9\frac{1}{2}$

11. If $3\sin 2\theta = 2\sin 3\theta$ and $0<\theta<\pi$, then value of $\cos\theta$ is :

(a) 1
(b) $\frac{1}{4}$
(c) 0
(d) None

12. The greatest positive integer, which divides $n(n+1)(n+2)(n+3)$ for all $n \in N$ is :

(a) 2
(b) 6
(c) 24
(d) 120

13. If $x^n - 1$ is divisible by $x - \lambda$, then the least positive integral value of λ is :

(a) 1
(b) 2
(c) 3
(d) 4

14. If $Z=\overline{Z}$, where Z is a complex number, then

(a) Z is purely real

(b) Z is purely imaginary

(c) Real part of $Z=$ imaginary part of Z

(d) Z is any complex number

15. $i^{57}+\frac{1}{i^{125}}$, when simplified has the value :

(a) 0 (b) $2i$ (c) $-2i$ (d) 2

16. The value of $(\sqrt{-1})(\sqrt{-1})$ is :

(a) 1 (b) -1

(c) $\frac{1}{2}$ (d) 0

17. The complex number $\frac{1+2i}{1-i}$ lies in/on.

(a) first quadrant (b) on x-axis

(c) Third quadrant (d) II[nd] quadrant

18. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n=-1$ is :

(a) 1 (b) 2

(c) 3 (d) 4

19. The values of x which satisfy both the inequalities $x-1\leq 0$ and $4x+3\geq 0$ lie in :

(a) $\left[\frac{-3}{4},1\right]$ (b) $(-\infty,1)$

(c) $(1,2)$ (d) $\left(\frac{-3}{4},1\right)$

20. For what values of x, the given inequality satisfy $\frac{1}{x-2}<0$:

(a) $x>2$ (b) $x>-2$

(c) $x<-2$ (d) $x<2$

(SECTION : B)

21. If $A=\{x : x$ is a natural number$\}$, $B=\{x : x$ is an even natural number$\}$, $C=\{x : x$ is an odd natural number$\}$ and $D=\{x : x$ is a prime number$\}$, find

(i) $A \cap D$ (b) $B \cap C$

22. Find the range of function $f(x) = x^2 + 2$, x is a real number.

23. Find the value of $\tan\left(\frac{19\pi}{3}\right) + \cot\left(\frac{-15\pi}{4}\right)$.

OR

Find the value of $\sin\left(\frac{17\pi}{4}\right) + \sin\left(\frac{-15\pi}{4}\right)$.

24. Prove that $2^{3n} - 1$ is divisible by 7 for all $n \in N$.

OR

Prove that $7^n - 3^n$ is divisible by 4 for all $n \in N$.

25. Find modulus and argument of a complex number $-1 - i$.

26. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

(SECTION : C)

27. A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like produce B. What is the least number that must have liked both products ?

OR

In a survey of 60 people it was found that 25 people read newspaper H, 26 read news paper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspaper. Find (i) The number of people who read at least one newspaper (ii) The number of people who read exactly one new paper.

28. Find domain and range of the following

(i) $f(x) = \sqrt{9 - x^2}$ (ii) $f(x) = |x - 1|$

29. Prove that $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$.

OR

Find the general solution for, $\sin x + \sin 3x + \sin 5x = 0$

30. Prove that

$$\cos 2x \cos\frac{x}{2} - \cos 3x \cos\frac{x}{2} - \cos 3x . \cos\frac{9x}{2}.$$

OR

Prove that

$$\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$$

31. Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

32. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more then 4% but less then 6% boric acid of we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added.

(SECTION : D)

33. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ if $\sin x = \frac{1}{4}$, x in IInd quadrant.

34. Prove that $2.7^n + 3.5^n - 5$ is divisible by 24 for $\forall n \in N$.

35. If α and β are different complex number with $|\beta| = 1$ than find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

36. Solve the system of inequalities graphically.

$$4x + 3y \le 60,\ y \ge 2x,\ x \ge 3,\ x, y \ge 0$$

OR

$$x + 2y \le 10,\ x + y \ge 1,\ x - y \le 0,\ x \ge 0, y \ge 0$$

Roll No.	
Name	
Class & Section	

Set

APEEJAY EDUCATION SOCIETY

APEEJAY COMMON ANNUAL EXAMINATION, 2019-20

MATHEMATICS

Time Allowed : 3.00 Hrs. **Class – XI** **Maximum Marks : 80**

General Instructions :

(i) All the questions are compulsory.

(ii) The question paper consists of 36 questions divided into 4 sections A, B, C and D.

(iii) Section-A comprises of 20 questions of 1 mark each.

Section-B comprises of 6 questions of 2 marks each.

Section-C comprises of 6 questions of 4 marks each.

Section-D comprises of 4 questions of 6 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each.

You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section-A

1. If A and B are two disjoint sets, then $n(A \cup B)$ is equal to

(a) $n(A)+n(B)$ (b) $n(A)+n(B)-n(A \cap B)$

(c) $n(A)+n(B)+n(A \cap B)$ (d) $n(A).n(B)$

2. The value of $(1+i)(1+i^2)(1+i^3)(1+i^4)$ is

(a) 2 (b) 0

(c) 1 (d) –1

3. If $|x+2| \leq 9$, then

(a) $x \in (-7, 11)$ (b) $x \in [-11, 7]$

(c) $x \in (-\infty, -7)$ (d) $x \in (-\infty, 7) \cup (11, \infty)$

4. A coin is tossed. If head comes up, a dice is thrown, but if tail comes up, the coin is tossed again. Find the probability of getting head and an even number.

(a) $\frac{7}{8}$ (b) $\frac{3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{3}{8}$

5. The distance of the point (3, 3, 4) from x-axis is

(a) $\sqrt{22}$ (b) 3

(c) 5 (d) 4

6. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Then the number of subsets of A containing exactly two elements is

(a) 20 (b) 40

(c) 45 (d) 90

7. The sum to infinity of the following series

6, 1.2, 0.24, ... is

(a) 7 (b) 7.7

(c) 7.5 (d) 6.5

8. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$ is equal to

(a) $2 \sin x$ (b) $\cos 2x$

(c) $\tan 3x$ (d) $\operatorname{cosec} 2x$

9. Distance between the straight lines $6x + 8y + 15 = 0$ and $3x + 4y + 9 = 0$ is

(a) $\frac{3}{10}$ units (b) $\frac{5}{13}$ units

(c) $\frac{2}{7}$ units (d) $\frac{4}{11}$ units

10. Let R be a relation on N, defined by $x + 2y = 8$. The domain of R is

(a) {1, 2, 3, 4} (b) {2, 4, 6, 8}

(c) {2, 4, 8} (d) {2, 4, 6}

11. The foci of the conic $2x^2 - 3y^2 = 5$ are .... & ....

12. If 6 boys and 6 girls sit in a row at random, then the probability that all the girls sit together is ....

13. In the expansion of $\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18}$, term number .... is independent of x.

OR

The coefficient of x^{-3} in the expansion of $\left(x-\frac{m}{x}\right)^{11}$ is ....

14. If the sum of n terms of an A.P. is $3n^2+5n$, then 164 is .... term.

15. If the arcs of same length in two circles subtend angles 65° and 110° at the centre, then the ratio of the radii of the circles is ....

16. Find x and y if $(x+y, 2)=(3, 2x+y)$.

17. There are 12 points in a plane, out of which 5 points are collinear. Find the number of straight lines that can be drawn joining them.

OR

How many numbers are there betwen 100 and 1000 such that none of their digits is 7.

18. Write contrapositive of the statement 'If a number is divisible by 9, then it is divisible by 3'.

19. Write the negative of the following statements :

(a) All integers are not rational numbers.

(b) 6 is divisible by 2 and 3.

20. Find the equation of the circle which touches Y-axis and has centre at (2, 3).

OR

Find the equation of parabola with vertex at origin, symmetric with Y-axis and passing through (2, –3)

Section-B **(2 marks each)**

21. If $\cos\frac{x}{2}=-\frac{1}{\sqrt{10}}, \frac{\pi}{2}<\frac{x}{2}<\frac{3\pi}{4}$ find $\cos x$.

OR

Prove that : $\frac{\cos 8^\circ-\sin 8^\circ}{\cos 8^\circ+\sin 8^\circ}=\tan 37^\circ$

22. Find the domain and range of the following function :

$$f(x)=\sqrt{16-x^2}$$

23. Differentiate $\dfrac{x^2+x\sin x}{x+\cos x}$ w.r.t. x.

OR

Differentiate $f(x)=x^3+27$ by 1^{st} principle.

24. Find the probability of atmost two tails or at least two heads in a toss of three coins.

25. Evaluate :

$$\lim_{x\to\pi/2}\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)\cos x}.$$

26. Evaluate :

$$\frac{\cot^2 15^\circ-1}{\cot^2 15^\circ+1}$$

Section-C **(4 marks each)**

27. Prove that $7^{2n}+2^{3n-3}.3^{n-1}$ is divisible by 25 for all $n\in N$ by mathematical induction.

OR

For all $n\in N$, prove that :

$$3.2^2+3^2.2^3+3^3.2^4+......+3^n.2^{n+1}=\frac{12(6^n-1)}{5}$$

28. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all the newspapers. Find :

(i) The number of people who read at least one of these three newspapers.

(ii) The number of people who read exactly one newspaper.

29. Find n in the bionomial expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, if the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is $\frac{1}{6}$.

30. The sum of three numbers in a G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an A.P. Find the numbers.

31. Solve the following system of inequations graphically

$$4x+3y\leq 60, \quad y\geq 2x, \quad x\geq 3, \; y\geq 0$$

OR

A manufacturer has 600 litres of 2% solution of acid. How many litres of 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%.

32. Find modulus, argument, multiplicative inverse and polar form of : $z=\frac{1}{1+i}$.

Section-D (6 marks each)

33. (a) The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12 and 14, find the remaining two observations.

(b) The probability that a person will get an electric contract is $\frac{2}{5}$ and the probability that he will not get plumbing contract is $\frac{4}{7}$. If the probability of getting at least one contract is $\frac{2}{3}$, what is the probability that he will get both the contracts. (4+2)

OR

(a) Find the mean and variance for the following distribution :

Class interval	Frequency
30-40	3
40-50	7
50-60	12
60-70	15
70-80	8
80-9	3
90-100	2

(b) Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that you both enter in the same section. **(4+2)**

34. (a) Find the distance of the point $(2, 5)$ from the line $3x + y + 4 = 0$ measured parallel to the line $3x - 4y + 8 = 0$.

(b) A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end point A lies on X axis and the end point B lies on Y axis. A point P is taken on the rod in such a way that $AP = 6$cm. Show that the locus of P is an ellipse. Find its eccentricity. **(3+3)**

OR

(a) The line $2x - 3y - 4 = 0$ is perpendicular bisector of the line segment AB and the co-ordinate of A is $(-3, 1)$. Find the co-ordinate of point B.

(b) An arc is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arc at a point 1.5 cm from one end. **(3+3)**

35. (a) Find the sum of n terms of the series

$$3 + 15 + 35 + 63 + \ldots\ldots\ldots\ldots$$

(b) If $\tan\beta = \dfrac{\tan\alpha + \tan\gamma}{1 + \tan\alpha \,.\tan\gamma}$

then prove that $\sin 2\beta = \dfrac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \,.\sin 2\gamma}$ **(4+2)**

36. (a) Solve the following equation for x

$$\sqrt{3}\cos x + \sin x = \sqrt{2}$$

(b) Prove that :

$$\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$$ **(3+3)**