Apeejay Common Pre-Board Examination
Class XII
Session 2022-23
Mathematics (Code - 041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**4 marks each**)** with sub parts.

SECTION A
(Multiple Choice Questions, Each question carries 1 mark)

Q1. If A is matrix of order m × n and B is a matrix such that AB′ and B′A are both defined, then order of matrix B is

(A) m × m (B) n × n (C) n × m (D) m × n

Q2. If matrix A = $[a_{ij}]_{2*2}$, where $a_{ij} = 1$ if $i \neq j$ & $a_{ij} = 0$ if i = j then A^2 is equal to

(A) I (B) A (C) 0 (D) None of these

Q3. The vectors from origin to the points A and B are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ respectively, then the area of triangle OAB is

(A)340 (B) $\sqrt{25}$ (C) $\sqrt{229}$ (D) $\frac{\sqrt{229}}{2}$

Q4. The derivative of $\tan x^o$ at $x = 45^o$ is

(A) $\frac{\pi}{180^o}$ (B) $\frac{2\pi}{180^o}$ (C) $\frac{180^o}{\pi}$ (D) $\frac{180^o}{2\pi}$

Q5. $\int e^x(\cos x - \sin x)dx$ is equal to

(A) $e^x \cos x + C$ (B) $e^x \sin x + C$

(C) $-e^x \cos x + C$ (D) $-e^x \sin x + C$

Q6. The order and degree of the differential equation $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^{1/4} + x^{1/5} = 0$ respectively are

(A) 2 and not defined (B) 2 and 2 (C) 2 and 3 (D) 3 and 3

Q7. The optimal value of a LPP may or may not exist if feasible region of the LPP is

(A) bounded (B) unbounded (C) closed (D) not defined

Q8. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a}-\vec{b}$ to be a unit vector?

(A) 30° (B) 45° (C) 60° (D) 90°

Q9. The value of integral $\int_{1/3}^{1}\frac{(x-x^3)^{1/3}}{x^4}dx$ is

(A) 6 (B)0 (C) 3 (D) 4

Q10. If the matrix A is both symmetric and skew symmetric, then

(A) is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Q11. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F = 4x + 6y be the objective function. The Minimum value of F occurs at

(A) (0, 2) only

(B) (3, 0) only

(C) the mid point of the line segment joining the points (0, 2) and (3, 0) only

(D) any point on the line segment joining the points (0, 2) and (3, 0).

Q12. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then value of x is

(A) 3 (B) ± 3 (C) ± 6 (D) 6

Q13. The points (a + 5, a – 4), (a – 2, a + 3) and (a, a) lie on a straight line for

(A) a = 0 (B) a= 1 (C) any value of a (D) no value of a

Q14. If P(A) = 0.4, P(B) = 0.8 and P(B | A) = 0.6, then P(A ∪B) is equal to

(A) 0.24 (B) 0.3 (C) 0.48 (D) 0.96

Q15. Solution of $\frac{dy}{dx}-y=1$, y (0) = 1 is given by

(A) $xy = -e^{x}$ (B) $xy = -e^{-x}$ (C) xy = – 1 (D) $y = 2e^{x} - 1$

Q16. If $y=\sqrt{\sin x+y}$, then $\frac{dy}{dx}$ is equal to

(A) $\dfrac{\cos x}{2y-1}$

(B) $\dfrac{\cos x}{1-2y}$

(C) $\dfrac{\sin x}{1-2y}$

(D) $\dfrac{\sin x}{2y-1}$

Q17. The value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is

(A) 0 (B) -4 (C) 3/2 (D) – 5/2

Q18. The equation of x-axis in space is

(A) x = 0, y = 0 (B) x = 0, z = 0 (C) x = 0 (D) y = 0, z = 0

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement ofReason (R). Choose the correct answer out of the following choices.

A. Both A and R are true and R is the correct explanation of A.
B. Both A and R are true but R is not the correct explanation of A.
C. A is true but R is false.
D. A is false but R is true.

Q19. *Assertion(A)*: The principal value of $\sin^{-1}\left(-\dfrac{1}{2}\right)$ is $-\dfrac{\pi}{6}$.

Reasoning(R): The principal value of $\sin^{-1}(-x)$ is $-\sin^{-1}(x)$ if $x \in [-1,1]$.

Q20. *Assertion(A)*: Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} - \hat{k}$ be two vectors. Angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is 90°

Reasoning (R): Projection of $\vec{a} + \vec{b}$ on $\vec{a} - \vec{b}$ is zero.

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

Q21. Evaluate $\sin^{-1}\left[\cos\left(\sin^{-1}\dfrac{\sqrt{3}}{2}\right)\right]$

OR

Find the value of $\sin\left[2\cot^{-1}\left(\dfrac{5}{12}\right)\right]$ □ □ □ □ □ □□□□□

Q22. A man of height 2 metre walks at a uniform speed of 5 km/h away from a lamp post which is 6 metre high. Find the rate at which the length of his shadow increases.

Q23. For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$,find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

OR

Find the direction cosines of a line which makes equal angles with the coordinate axes.

Q24. Prove that the function f defined by

$$f(x)=\begin{cases}\dfrac{x}{|x|+2x^2}, & x\neq 0\\ k, & x=0\end{cases}$$

remains discontinuous at x = 0, regardless the choice of k.

Q25. Three vectors $\vec{a},\vec{b}\,\&\,\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, if $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=2$.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each.)

Q26.Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed. 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

OR

Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

Q27. Solve the following Linear Programming Problem graphically: Max.Z=10500x+9000y subject to

$x+y \leq 50$

$20x+10y \leq 800$

$x,y \geq 0$

Q28. Find a particular solution of the differential equation $\frac{dy}{dx}+y\cot x=4x\,\mathrm{cosec}\,x$ $(x\neq 0)$, given that y = 0 when $x=\pi/2$.

OR

Solve the differential equation : $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$

Q29. Evaluate $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$

OR

Evaluate $\int_0^{\pi/4}\log(1+\tan x)dx$

Q30. If $\int\frac{\cos 8x+1}{\tan 2x-\cot 2x}dx=\lambda\cos 8x+c$, then find the value of λ.

Q31. Evaluate $\int\frac{2x}{(x^2+1)(x^2+3)}dx$

SECTION D

(This section comprises of long answer type questions (LA) of 5 marks each.)

Q32. Let A = {1, 2, 3, ... 9} and R be the relation in A ×A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A ×A. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)]

OR

Let f : N → N be defined by $f(n)=\begin{cases}\frac{n+1}{2}, odd\\ \frac{n}{2}, even\end{cases}$ for all $n \in N.$

check whether the function f is one-one & onto and hence bijective or not. Justify your answer.

Q33. Draw a rough sketch of the given curve y = 1 + |x +1|, x = –3, x = 3, y = 0 and find the area of the region bounded by them, using integration.

Q34. Given $A=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$, $B=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ find BA and use this to solve the system of equations y + 2z = 7, x – y = 3, 2x + 3y + 4z = 17.

Q35. Find the shortest distance between lines $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = 4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

OR

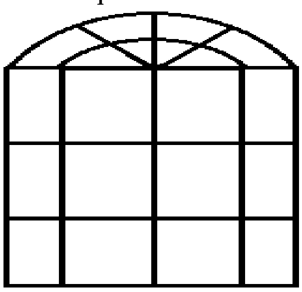
Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines:

$\frac{(x-8)}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{(x-15)}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

Q.36 Read the following passage and answer the questions that follow.



Angad is having a window in his house which is in the form of a rectangle surmounted by a semicircular opening. He takes the height of the rectangular portion of the window to be x units and radius of the semicircular portion to be r units and found that the total perimeter of the window is 10 m.

(i) Find the area of the given window, using the above dimensions in terms of x and r.

(ii) What should be the value of r, when area of the window is maximum?

(iii) Angad is interested in finding the dimensions of the window to admit maximum light through the whole opening, find the dimensions of the rectangle.

OR

(iii) Find the area of window to admit maximum light.

Q37. Read the following passage and answer the questions that follow.

Lakshay and Sukrit are students of class XII (Non-Medical). They are good friends and play basketball in the school team. One day when they were bouncing a basketball on the ground, they noticed that the basketball follows some increasing and decreasing patterns. To clear the concepts of increasing and decreasing functions and maxima and minima they initiated a discussion by taking a function $f(x) = \sin^4 x + \cos^4 x$, where $0 \le x \le \pi/2$.

(i)What is the critical value of the above function in the interval $0 \le x \le \pi/2$.

(ii)Write the conditions for a function to be strictly increasing and strictly decreasing on an open interval.

(iii)Find the intervals on which the function $f(x) = \sin^4 x + \cos^4 x$, where $0 \le x \le \pi/2$ is (a)strictly increasing (b)strictly decreasing.

OR

(ii) If the function is taken as $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [1, 5]. Find the absolute maximum and absolute minimum values.

Q38. In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability 1/3. Let E_1, E_2, E be the events that the Student knows the answer, guesses the answer and answers correctly respectively.

(i) What is the total probability that answer is correct?

(ii) What is the probability that the student knows the answer given that he answered it correctly?