General Instructions:

1. All questions are compulsory.

2. The question paper consists of 29 questions divided into three sections A, B and C. Section-A comprises of 10 questions of one mark each, Section-B comprises of 12 questions of four marks each and Section-C comprises of 7 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted.

Section-A

1. A binary operation * on Z is defined by \( a * b = a + b - 4 \), for all \( a, b \in Z \). Find the identity element for * on Z.

2. If \( \tan^{-1}(\cot \theta) = 20 \), find the value of \( \theta \).

3. Find the value of \( x \) for which the angle between \( \vec{a} = 2xi + 4xj + k \) and \( \vec{b} = 7i - 2j + k \) is obtuse.

4. If \( A \) is a matrix of order 2 \( \times \) 3 and \( B \) is a matrix of order 3 \( \times \) 5, what is the order of \((AB)^T\).

5. If \( A \) is a square matrix of order 3 and \( |A| = 5 \) find \( |A \cdot \text{adj } A| \).

6. Find the value of \( \int_0^{\pi/2} \log \frac{3 + 5 \cos x}{3 + 5 \sin x} \, dx \)

7. Find the area of the parallelogram whose adjacent sides are given by the vectors \( \vec{a} = 3i + j + 4k \) and \( \vec{b} = i - j + k \).

8. If \( A^T = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix} \) and \( B = \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix} \) find \((A + 2B)^T\).

R/4
9. Find \[ \int \frac{1 + \cot x}{x + \log \sin x} \, dx \]

10. Find the direction cosines of the vector which passes through the points \((1, 4, -7)\) and \((2,6, -4)\).

Section-B

11. Show that the relation \(R\) on the set \(A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}\) given by
\[ R = \{(a, b) : |a - b| \text{ is a multiple of 4}\} \]

is an equivalence relation. Also find all the elements related to 2.

OR

Show that the function \(f : R \rightarrow R\) given by \(f(x) = 7x - 3\) is a bijective function. Also write the inverse of \(f\).

12. Simplify : \(\sin^{-1} \left( \frac{5}{13} \cos x + \frac{12}{13} \sin x \right)\)

13. Using the properties of determinants, prove the following :
\[
\begin{vmatrix}
3a & -a + b & a + c \\
3a & 3b & c - b \\
a - c & b - c & 3c
\end{vmatrix} = 3(a + b + c)(ab + bc + ca)
\]

14. Evaluate : \(\int \frac{3x - 1}{(x - 1)(x - 2)(x - 3)} \, dx\)

15. If \(f(x) = \begin{cases} a + b & \text{if } x < 6 \\ \frac{x - 6}{|x - 6|} & \text{if } x > 6 \end{cases}\) is a continuous function at \(x = 6\), find the values of \(a\) and \(b\).

16. If \(\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)\) then show that \(\frac{dy}{dx} = \frac{1 - y^2}{\sqrt{1 - x^2}}\)

17. Differentiate with respect to \(x\) : \(x^\sin x + (\sin x)^\cos x\).

OR

Using Rolles theorem find the points on the curve \(y = x^2 + 5x + 6, \ x \in [-3, -2]\) where the tangent is parallel to the x-axis.
18. A line makes angles $\alpha, \beta, \gamma$ and $\delta$ with the diagonals of a cube, prove that 
\[ \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3} \]

19. Solve the differential equation \( (x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1} \).

OR

Solve: \( \left( 1 + e^y \right) dx + e^y \left( 1 - \frac{x}{y} \right) dy = 0 \)

20. If the sum of two unit vectors is a unit vectors show that the magnitude of their difference is $\sqrt{3}$.

OR

If \(\vec{a}, \vec{b}\) and \(\vec{c}\) are three vectors such that \( |\vec{a}| = 3, \ |\vec{b}| = 4 \) and \( |\vec{c}| = 5 \) and each one of them is perpendicular to the sum of the other two, then find \( |\vec{a} + \vec{b} + \vec{c}| \).

21. Two dice are thrown simultaneously. Let \(X\) denotes the number of sixes. Find the probability distribution of \(X\). Also find the mean and variance of \(X\), using the probability distribution table.

22. Form the differential equation of the family of circles touching the \(x\)-axis at the origin.

Section-C

23. If \(A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \), find \(A^{-1}\) and hence solve the following system of equations:

\[
2x + 3y + z = 11 \\
-3x + 2y + z = 4 \\
5x - 4y - 2z = -9
\]

OR

Find the inverse of the matrix \( \begin{bmatrix} 1 & 3 & -2 \\ 3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} \) if exists, using elementary row transformation.
24. Find the vector as well as Cartesian equation of the planes through the intersection of planes \( \mathbf{r} \cdot (2\mathbf{i} + 6\mathbf{j}) + 12 = 0 \) and \( \mathbf{r} \cdot (3\mathbf{i} - \mathbf{j} + 4\mathbf{k}) = 0 \), which are at a unit distance from the origin.

25. Evaluate: \( \int \frac{x + 2}{(x^2 + 3x + 3) \sqrt{x + 1}} \, dx \)

OR

Evaluate: \( \int_1^2 (x^2 + x + 1) \, dx \) as limit of a sum.

26. A diet for sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods A and B are available at a cost of Rs. 4/- and Rs. 3/- per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of mineral and 40 units of calories whereas one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of A and B should be used to have least cost satisfying the requirements.

27. Find the area bounded by \( y = x^2 + 1 \), \( y = x \), \( x = 1 \) and \( y = 2 \).

28. A bag contains four balls. Two balls are drawn at random and are found to be white. What is the probability that all balls are white?

29. At 8 A.M. ship A is 65 km due east of another ship B. Ship A is then sailing due west at 10 km/hr and ship B is sailing due south at 15 km/hr. If they continue sailing on their respective paths, when will they be nearest to one another, and how near?