FIRST TERM EXAMINATION, 2012–2013
SKT-50
MATHEMATICS

Time Allowed : 3 hours
CLASS–XII M.M. : 100

General Instructions :

(1) All questions are compulsory.
(2) Section A consist of 10 questions of 1 mark each.
(3) Section B consist of 12 questions of 4 marks each.
(4) Section C consist of 7 questions of 6 marks each.

SECTION–A

1. Write the value of \( \int_{0}^{2} x \lfloor x \rfloor \, dx \).

2. Write the value of \( \int \frac{x^{a-1} + e^{x-1}}{x^a + e^x} \, dx \).

3. If \( \int (e^{ax} + bx) \, dx = 4e^{ax} + \frac{3x^2}{2} \), find the values of ‘a’ and ‘b’.

4. Evaluate : \( \int_{0}^{1} \frac{1}{\sqrt{5x+3}} \, dx \).

5. What is the degree of the differential equation \( \frac{d^2 y}{dx^2} = \sqrt{\left( \frac{dy}{dx} \right)^2} \)?

6. Write the integrating factor of the following differential equation.
   \( x \log x \frac{dy}{dx} + y = 2 \log x \)

7. If \( f(x) = \sin x^a \), find \( \frac{dy}{dx} \).

8. What is the value of ‘c’, if mean value theorem holds for the function \( f(x) = x (x - 2) \), \( c \in [1, 2] \)?

9. Write the set where \( f(x) = \tan x \) is discontinuous.

10. Find the maximum value of \( \sin x + \cos x \).

SECTION–B

11. Let \( f(x) = x \lfloor x \rfloor \), \( \forall \, x \in \mathbb{R} \). Discuss the derivability of \( f(x) \) at \( x = 0 \).

12. Find the value of the constant ‘k’, so that the function ‘f’ defined below is continuous at \( x = 0 \) where

P.T.O.
13. Show that the function $f(x) = \tan^{-1} (\sin x + \cos x)$, $x > 0$ is always an strictly increasing function in $(0, \pi/4)$.

14. Evaluate:

$$\int \frac{x^2 - 1}{x^2 + x} \, dx$$

15. Evaluate:

$$\int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x}) \, dx$$

16. Evaluate: $\int \sin (\log x) \, dx$.

17. Evaluate: $\int_0^{\pi/2} \frac{dx}{2 \cos x + 4 \sin x}$.

18. Solve the following differential equation:

$$(1 + y + x^2 y) \, dx + (x + x^3) \, dy = 0$$

where $y = 0$ when $x = 1$

19. Solve:

$$\left( x \cos \frac{y}{x} + y \sin \frac{y}{x} \right) y \, dx = \left( y \sin \frac{y}{x} - x \cos \frac{y}{x} \right) x \, dy.$$

20. Evaluate:

$$\int \sqrt{\frac{1 - \sqrt{x}}{1 + \sqrt{x}}} \, dx$$

21. Using integration, find the area of the region:

$$\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}, x \in R, y \in R\}$$

22. Due to severe flood conditions in Assam region, government decided to distribute food packets using helicopters. A helicopter is flying along curve $y = x^2 + 7$. And a group of stranded people are placed at a point $(3, 7)$. The pilot has to maneuver the helicopter to drop the food packets.

(a) Find the nearest distance between the helicopter and group of people. (3)

(b) What values are exhibited by the pilot while doing this work? (1)

SECTION—C

23. Ram Kishan had a square plot bounded by the lines $x = 0$, $x = 4$, $y = 4$ and $y = 0$. He decided to donate $1/3$ of his land to an NGO to build a school. His two sons objected and asked their father to reduce the donated
land area. Ram Kishan then decided to divide his land by drawing two curves in it given as \( y^2 = 4x \) and \( x^2 = 4y \). The shape of the divided land was such, that his sons thought their father had agreed to them. But, in fact Ram Kishan had given 1/3 of his land to NGO.

(a) Prove that (using integration) the curves \( y^2 = 4x \) and \( x^2 = 4y \) divide the area of the square land bounded by \( x = 0 \), \( x = 4 \), \( y = 5 \) and \( y = 0 \) into three equal parts.

(4)

(b) What values Ram Kishan had displayed?

(2)

24. An isosceles triangle of vertical angle 2\( \theta \) is inscribed in a circle of radius ‘a’. Show that the area of the triangle is maximum when \( \theta = \pi/6 \).

25. Find the angle of intersection of the curves \( y^2 = 4ax \) and \( x^2 = 4by \).

26. Find : \( \int \frac{x^2}{x^4 + x^2 - 2} \, dx \).

27. Evaluate : \( \int_0^1 (x^2 - x) \, dx \) as a limit of the sum.

28. Evaluate : \( \int_{-1}^{3/2} |x \sin (\pi x)| \, dx \).

29. Evaluate : \( \int_0^{\infty} \frac{x \tan x}{\sec x + \tan x} \, dx \).